

Chapter 06 연습문제 정답

1.

$$(a) f_X(x) = \begin{cases} \frac{1}{2}, & x = 0, 1 \\ 0, & \text{다른 곳에서} \end{cases} \quad f_Y(y) = \begin{cases} \frac{1}{12}, & y = 0 \\ \frac{1}{4}, & y = 1, 3 \\ \frac{5}{12}, & y = 2 \\ 0, & \text{다른 곳에서} \end{cases}$$

(b) $\frac{1}{3}$

(c) $E(X) = \frac{1}{2}, \quad E(Y) = \frac{11}{6}$

(d) $Var(X) = \frac{1}{2}, \quad Var(Y) = \frac{29}{36}$

(e) $-\frac{1}{4}$

2.

$$(a) f_X(x) = \begin{cases} \frac{3}{20}, & x = 1 \\ \frac{13}{60}, & x = 2 \\ \frac{17}{60}, & x = 3 \\ \frac{7}{20}, & x = 4 \\ 0, & \text{다른 곳에서} \end{cases} \quad f_Y(y) = \begin{cases} \frac{1}{5}, & y = 1 \\ \frac{7}{30}, & y = 2 \\ \frac{4}{15}, & y = 3 \\ \frac{3}{10}, & y = 4 \\ 0, & \text{다른 곳에서} \end{cases}$$

(b) $\frac{1}{4}$

(c) $E(X) = \frac{17}{6}, \quad E(Y) = \frac{8}{3}$

(d) $Var(X) = \frac{41}{36}, \quad Var(Y) = \frac{11}{9}$

(e) $-\frac{1}{4}$

3.

(a) $\frac{1}{2}$

(b) $\frac{5}{24}$

(c) $\frac{5+8\sqrt{2}}{40}$

(d) $\frac{2}{3}$

4.

(a) $f_X(x) = \frac{1}{2}(3x^2 + 1), 0 < x < 1$

$f_Y(y) = \frac{1}{2}(3y^2 + 1), 0 < y < 1$

(b) $E(X) = E(Y) = \frac{5}{8}$

(c) $Var(X) = Var(Y) = \frac{73}{960}$

(d) $-\frac{1}{64}$

(e) $-\frac{15}{73}$

5.

(a) $f_X(x) = 3x^2, 0 < x < 1$

$f_Y(y) = 1 + 2y - 3y^2, 0 < y < 1$

(b) $\frac{1}{2}$

(c) $E(X) = \frac{3}{4}, E(Y) = \frac{5}{12}$

(d) $Var(X) = \frac{3}{80}, Var(Y) = \frac{43}{720}$

(e) $\frac{1}{48}$

(f) $\frac{5}{\sqrt{129}}$

6.

$$(a) f_X(x) = \begin{cases} 0.935, & x=0 \\ 0.048, & x=1 \\ 0.017, & x=2 \\ 0, & \text{다른 곳에서} \end{cases} \quad f_Y(y) = \begin{cases} 0.94, & y=0 \\ 0.03, & y=1 \\ 0.023, & y=2 \\ 0.007, & y=3 \\ 0, & \text{다른 곳에서} \end{cases}$$

(b) 0.956

(c) $E(X) = 0.048, \quad E(Y) = 0.097$

(d) $Var(X) \approx 0.0457, \quad Var(Y) \approx 0.1756$

(e) 0.0573

(f) 약 0.6396

7.

(a)

$X \backslash Y$	1	2	3	4	$f_Y(y)$
1	0.24	0.15	0.11	0.07	0.57
5	0.12	0.04	0.04	0.05	0.25
10	0.1	0.06	0.02	0.00	0.18
$f_X(x)$	0.46	0.25	0.17	0.12	1

(b) 0.55

(c) $E(X) = 1.95, \quad E(Y) = 3.62$

(d) $Var(X) = 1.1075, \quad Var(Y) = 11.7156$

(e) -0.509

(f) 약 0.1413

8.

(a)

$$f_X(x) = \frac{10^x}{x!} e^{-10}$$

$$f_Y(y) = \frac{3^y}{y!} e^{-3}$$

(b) X 와 Y 는 독립이다.

$$(c) f(x|1) = \frac{10^x}{x!} e^{-10}, \quad x = 0, 1, 2, \dots$$

(d) 약 0.0001

9.

(a) $f_X(x) = 1 + 3x^2 - 4x^3, 0 < x < 1$

$f_Y(y) = 4y^3, 0 < y < 1$

(b) X 와 Y 는 독립이 아니다.

(c) $E(X) = \frac{9}{20}, E(Y) = \frac{4}{5}$

(d) $Var(X) = \frac{77}{1200}, Var(Y) = \frac{2}{75}$

(e) $\frac{3}{200}$

(f) 약 0.3626

10.

(a) $f(x, y) = \frac{4}{17}, 8.8 \leq x \leq 10.5, 6 \leq y \leq 8.5$

(b) $f_X(x) = \frac{10}{17}, 8.8 \leq x \leq 10.5$

$f_Y(y) = \frac{2}{5}, 6 \leq y \leq 8.5$

(c) $F(x, y) = \frac{4}{85}(5x - 44)(y - 6)$

(d) $F_X(x) = \frac{2}{17}(5x - 44), 8.8 \leq x \leq 10.5$

$F_Y(y) = \frac{2}{5}(y - 6), 6 \leq y \leq 8.5$

(e) X 와 Y 는 독립이다.

(f) 약 0.02

11.

(a) $k = \frac{3}{5000}$

(b) $f_X(x) = \frac{1}{50}e^{-x/50}, x > 0$

$f_Y(y) = \frac{3}{100}e^{-3y/100}, y > 0$

(c) X 와 Y 는 독립이다.

(d) $F(x, y) = (1 - e^{-x/50})(1 - e^{-3y/100}), x > 0, y > 0$

(e) $f(x|y) = \frac{1}{50}e^{-x/50}, x > 0$

(f) $(1 - e^{-1/5})(1 - e^{-3/20}) \approx 0.02525$

12.

(a) $f_X(x) = \frac{3}{4}x(2-x), 0 < x < 2$

$$f_Y(y) = \frac{3}{8}(y-2)^2, 0 < y < 2$$

(b) X 와 Y 는 독립이 아니다.

(c) $f(y|1) = 1, 0 < y < 1$

(d) $\frac{1}{12}$

(e) $\frac{3}{8}$

(f) $\frac{1}{2}$

13.

(a) $f_X(x) = -\ln x, 0 < x < 1$

$$f_Y(y) = 1, 0 < y < 1$$

(b) X 와 Y 는 독립이 아니다.

(c) $f(y|x) = -\frac{1}{y \ln x}, 0 < x < y < 1$

(d) $\frac{4(-1 + \ln 3)}{9(\ln 3)^2} \approx 0.0363$

(e) $\ln 2$

(f) $1 - \frac{\ln 2}{\ln 3} \approx 0.3691$

14.

(a) -1

(b) 2

15.

(a) $f_X(x) = \frac{x}{2}, 0 < x < 2$

$$f_Y(y) = \frac{2}{25}y, 0 < y < 5$$

(b) $f(y|x) = \frac{2}{25}y, 0 < y < 5$

(c) $\frac{12}{25}$

(d) $\frac{25}{18}$

16.

(a) $f_X(x) = e^{-x}, x > 0$

$f_Y(y) = \frac{1}{(1+y)^2}, y > 0$

(b) X 와 Y 는 독립이 아니다.

(c) $f(y|x) = x e^{-xy}, x > 0, y > 0$

(d) $\frac{1}{x^2}$

17.

(a) $f_X(x) = \frac{10}{3}x(1-x^3), 0 < x < 1$

$f_Y(y) = 160y^4, 0 < y < 1/2$

(b) $f(y|1/2) = \frac{192}{7}y^2, \frac{1}{4} < y < \frac{1}{2}$

(c) $\frac{37}{189}$

(d) $\frac{291}{62720}$

18.

(a)

$X \backslash Y$	1	2	3	$f_Y(y)$
1	0.06	0.16	0.10	0.32
2	0.02	0.16	0.35	0.53
3	0.02	0.08	0.05	0.15
$f_X(x)$	0.10	0.40	0.50	1.00

(b)

Y	1	2	3
$P(Y = y)$	0.32	0.53	0.15

(c) $\mu_X = 2.3, \mu_Y = 1.77$

(d) $Var(X) = 3.61, Var(Y) = 0.6571$

(e) $\frac{0.379}{(1.9)(0.81)} \approx 0.2463$

(f) $E(Y|X = x) \approx 1.5285 + 0.105x$

19.

(a)

$\begin{matrix} X \\ Y \end{matrix}$	0	1	2	3	$f_Y(y)$
0	0.065	0.021	0.008	0.006	0.1
1	0.132	0.081	0.066	0.021	0.3
2	0.246	0.192	0.114	0.048	0.6
$f_X(x)$	0.443	0.294	0.188	0.075	1.0

(b)

X	0	1	2	3
$f_X(x)$	0.443	0.294	0.188	0.075

(c) $\mu_X = 0.895, \quad \mu_Y = 1.5$

(d) $Var(X) = 0.919975, \quad Var(Y) = 0.45$

(e) $\frac{0.379}{(0.959)(0.671)} \approx 0.589$

(f) $E(Y|X = x) \approx 1.131 + 0.412x$

20.

(a) $f_X(x) = 3(x-1)^2, \quad 0 < x < 1$

$f_Y(y) = 3(y-1)^2, \quad 0 < y < 1$

(b) $\mu_X = \mu_Y = \frac{1}{4}$

(c) $\sigma_X = \sigma_Y = \frac{\sqrt{3}}{4\sqrt{5}}$

(d) $-\frac{1}{80}$

(e) $-\frac{1}{3}$

(f) $E(Y|X = x) \approx 0.33325 - 0.333x$

21.

(a) $f_X(x) = 2x, \quad 0 < x < 1$

$f_Y(y) = 2(1-y), \quad 0 < y < 1$

(b) $f(y|x) = \frac{1}{x}, \quad 0 < y < x$

(c) $\frac{x}{2}$

(d) $\frac{x^2}{12}$

(e) $\frac{1}{24}$

(f) $\frac{1}{72}$

(g) $\frac{1}{18}$

(h) $\frac{1}{18}$

22. $\frac{9}{4}$